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# Book Review: *On Growth and Form (Fractal and Non-Fractal Patterns in Physics)*

**On Growth and Form (Fractal and Non-Fractal Patterns in Physics).** H. Eugene Stanley and Nicole Ostrowsky, Eds., Martinus Nijhoff, Boston/Dordrecht/Lancaster, 1986, 308 pp.

This volume is an account of the proceedings of a NATO ASI held in Cargèse, 26 June–6 July 1985. Part A, consisting of "The Course," contains 11 lectures, for about 60% of the volume, and Part B, "The Seminars," contains 22 lectures. This is a comfortable balance for a volume of this sort, and the book contains a lot of information, from pedagogical reviews to interesting new results.

The subject, of course, is a continuing topical one. The historical concerns of physics have been with the tractable geometry of the regular; but the computer is helping us to begin the exploration of the geometry of the irregular, the geometry of fractals. Ten years ago the word fractal did not exist; today, it is hard to find an issue of, e.g., *Physical Review Letters* without some discussion of fractals. Fractals are geometric objects which possess an infinite number of features of different sizes or, in other words, they possess no characteristic scale. The Weierstrass function, for example, has wiggles upon wiggles on all scales and is nowhere flat, which causes this everywhere-continuous function to be nowhere differentiable. Such mathematical examples used to be considered to be pathological and most certainly not to have any physical significance. Mandelbrot's classic 1967 paper in *Science*, "How Long is the Coast of Britain? Self-Similarity and Fractional Dimension," drastically changed this view. Indeed, the *physical* notion of nonintegral, or fractal, dimension can now properly be regarded as due to Mandelbrot.

A key missing idea in physics until now has been that of dimension. Thus, the answer to the question posed in Mandelbrot's title, as had been observed by Richardson about half a centruy earlier, depends—it depends on the size of the measuring rod used to perform the measurement. The length of the coastline "really" is infinite because the perimeter has dimension higher than one. That a suitable physical notion of dimension has



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been missing from our thinking is evidenced also in its past absence from writings on Brownian motion, certainly an old friend to physics, and certainly also the generator of fractal curves, whose dimension, in fact, is two! By now, there are many physical examples where fractal modeling has been discovered to be successful, including the surface of clouds, the infrared radiation intensity curve as a function of distance along a cloud, the surface of porous catalysts, soot particles, the space in which turbulence is concentrated, niobium–germanium cluster sputter deposited on quartz substrates, colloidal silica aggregates, a gel at the gelation point, polymers in critical phenomena, percolation clusters, invasion percolation patterns, viscous fingering, event times for electron hopping in amorphous materials, and on and on. After a fractal conference, one gets the feeling that it is difficult to find real objects which (over some range of sizes) are not fractal.

The Cargése Conference was organized by Gene Stanley and Nicole Ostrowsky, and the main question this proceedings volume seems to pose is, "How does Nature create this multitude of shapes that are fractal?" The idea of dimension receives a good deal of attention in the lectures by Gene Stanley, who discusses ten different, but not all independent, dimensions which can be useful for characterizing a random fractal. Antonio Coniglio shows that an infinite set of exponents can be generated to describe a fractal, which are related to the voltage distribution across a network of resistors with the same structure as the fractal. Leo Kadanoff reports on research in progress with several collaborators on how a complex object like a fractal may be specifically covered by a measure.

Paul Meakin's lecture on aggregation and growth simulations has numerous good figures. It is Meakin's values for dimensions for a variety of models that theorists strive to predict. Hans Hermann describes a variety of growth models in the spirit of critical phenomena, using the ideas of scaling laws. The differences between growth models and static models (such as the Ising model), both of which have fractional clusters, is also discussed. Alla Margolina discusses a variety of rules for growth on the perimeter of a cluster, which she terms butterfly, ant, and caterpiller rules. Deepak Dhar proves that even though the Eden cluster is basically circular in two dimensions, in some higher dimension it must be asymmetric.

Robin Ball analyzes the diffusion-limited aggregation (DLA) process and gives examples of DLA found in experiment, including the electrodeposition of copper in a copper sulfate solution, the electrodeposition of zinc metal in an interfacial layer, viscous fingering, and in patterns of dielectric breakdown. Tom Witten discusses variations of the DLA model, and raises suspicions that anisotropic properties of the DLA cluster may arise in larger size simulations than have so far been achieved.

One of the highlights of the volume is the paper of Lee Turkevich on calculating the fractal dimension of DLA and other structures. Turkevich notices that the DLA structure in 2D can be placed roughly into a square. He then makes an exact analogy between the DLA problem and the calculation of the electric field lines for a square held at a fixed potential enclosed by a surface at infinity. This model leads to useful new insights into why fractal dimension values typically found in more direct calculations, such as $\sim 5/3$ for DLA in 2D, are what they are. Hermann discusses epidemic models, where a cluster perimeter site is chosen at random either to die or to become a growth site and make its neighbors available for the same selection process. This generates a fractal structure which possesses holes of all scales and looks quite different from DLA.

Cluster–cluster aggregation and DLA models are described in theoretical and numerical work by Max Kolb and co-workers; while experimental work by Dave Cannell is also reported, using light scattering techniques for measurements of clusters of colloidal silica particles in NaCl. Dale Schaefer, also using light scattering techniques, has found several materials to have a fractal structure. José Teixeiras lectures on scattering experiments designed to probe fractal structures, e.g., of proteins and immunoglobin aggregates. Viscous fingering experiments with Hele-Shaw cells are described and discussed by Gerard Daccord, Johann Nittmann, and Gene Stanley.

The volume contains papers and lectures on many other topics. Mike Shlesinger, with Joe Klafter, talks about Lévy flights vs. Lévy random walks. (The celebrated Weierstrass function actually makes an appearance in this talk!) Dietrich Stauffer gives a nice review of percolation clusters. Francois Leyvraz describes a rate equation approach to aggregation phenomena.

David Landau discusses within a kinetic model the percolation and non-percolation-like properties of gelation. Fereydoon Family describes theoretically the time-dependent kinetics of aggregation models, using a dynamic scaling theory and the Smoluchowski equation. Etienne Guyon examines the relationship between flow and form in random materials. He points out many experimental results that are not explained by existing models. Jorge Willemsen also lectures on flow through porous media. He emphasizes how one fluid can displace another (invasion percolation) of a different viscosity. In the limit of an infinite viscosity ratio, DLA-like shapes appear. Jean-Pierre Boon analyzes the development and growth of patterns arising from reaction-diffusion models.

Other work reported on in the volume includes that on Wetting Induced ·Aggregation (Daniel Blysens *et al*), Light Scattering from Aggregating Systems (John G. Rarity *et al.*), Crack Propagation and the Onset of Failure (Sara Solla), Branched Polymers (Mohamed Daoud), Fractal Properties of Clusters during Spinodal Decomposition (Rashmi Desai and Alan R. Denton), Monte Carlo Study of Dendritic Growth (Jamos Kertész *et al.*), and theoretical work on the Theta Point (Nareem Jan *et al.*) and Field Theories of Walks and Epidemics (Luca Peliti).

It is interesting to reflect that a volume such as this, although it deals with physical phenomena and properties of materials that might have been studied half a century ago, could not have been written even 20 years ago. The idea of the fractal, helped along by Mandelbrot and by the computer, has invaded our thinking, and the geometry of the irregular is now widely perceived as an art of the possible in the modeling of many aspects of Nature. I wonder whether its mathematical Godfathers, such as Besicovitch, Cantor, Hausdorff, and Weierstrass, would be surprised at all this, or whether, instead, they would recognize something basically familiar.

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# Book Review: *An Introduction to Chaotic Dynamical Systems*

**An Introduction to Chaotic Dynamical Systems.** Robert L. Devaney. Benjamin/Cummings, 1986.

Professor Devaney has done a real service for the scientific community. In three chapters he has given a rigorous and correct, yet lucid and accessible, introduction to the important and fascinating subject of chaotic dynamical systems. Almost all of the material is concerned with (apparently) simple maps on $\mathbf{R}^1$, $\mathbf{R}^2$, $\mathbf{R}^3$, and in fact the first chapter, "One Dimensional Dynamics," takes up almost half of the book. Nevertheless, in the first two chapters many fundamental topics in dynamics are introduced, including hyperbolocity, symbolic dynamics, structural stability, stable and unstable manifolds, and Hopf bifurcations. Chapter 3 covers Julia sets. Throughout the book the writing is clear, the approach is modern, and there are plenty of exercises which really do illuminate the subject. Only a modest background in advanced calculus is required of the reader, and this is reviewed at the beginning of Chapter 1. This work can be recommended to researchers and students in the many fields where chaos seems to play an important role.

It might have been a nice idea for the author to include a few more connections with "real" physical systems, beyond the overview which is given early in Chapter 1. Maps of the circle, for example, provide a natural starting point for the study of the Poincaré map for the van der Pol oscillator. However, it must be recognized that the inclusion of more physics would have made the book longer and more advanced, and probably would have diminished its accessibility.

The text has been prepared using $\mathrm{T_EX}$, and so the mathematical notation is clear and attractive. Unfortunately, the reproduction process seems to have been defective. Perhaps toner should have been added to the laser printer! Many pages are quite faint and difficult to read. In my copy half of p. 52 is actually illegible. This is an easily corrected flaw in the production of an otherwise excellent book.

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